

# Improving wave model validation based on RMSE

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# Motivation

- This work has been developed during a tuning of WWIII in the Mediterranean sea.
- RMSE, NRMSE and SI provided unsatisfactory indication of wave model performances during a validation of WWIII in the Mediterranean sea in storm conditions.
- **A best fit of the model based on RMSE led to parametrizations affected by strong negative bias.**

# Conclusions

- RMSE, NRMSE and SI tend to be **systematically better for simulations affected by negative bias**. This is mostly evident when:
  - We are tuning parameters involving an amplification of the results.
  - Correlation coefficient appreciably smaller than 1 ,  $\rho < 0.9$ .
  - Standard deviation of the observations of the same order of the average,  $\sigma_o / \bar{O} \sim 1$  .
- The indicator HH introduced by Hanna and Heinold (1985), defined by:

$$HH = \sqrt{\frac{\sum (S_i - O_i)^2}{\sum S_i O_i}} = \sqrt{\frac{(S - O)^2}{SO}}$$

provides more reliable information, being minimum for simulations unaffected by bias.

# Validation of wave model in the Mediterranean sea

## Model Wavewatch III

- Arduin et al. (2010) source terms (41 parameterizations).
- Tolman and Chalikov (1996) source terms, not tuned to the Mediterranean sea conditions.

## Statistical indicators used for validation

Normalized Bias (NBI)

$$NBI = \frac{(\bar{S} - \bar{O})}{\bar{O}}$$

Root Mean Square Error

$$RMSE = \sqrt{(S - O)^2}$$

Normalized Root Mean Square Error (NRMSE)

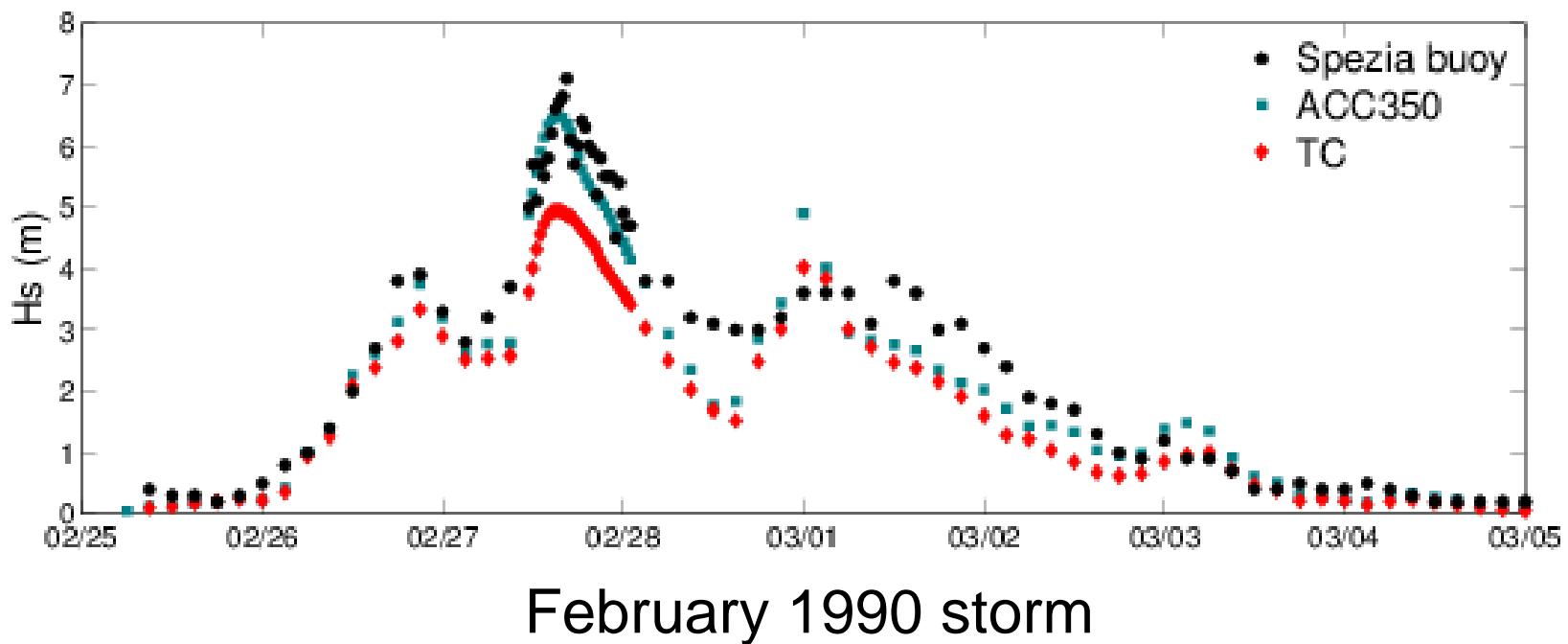
$$NRMSE = \sqrt{\frac{(S - O)^2}{\bar{O}^2}}$$

Correlation coeff. ( $\rho$ )

$$\rho = \frac{\sum (S_i - \bar{S})(O_i - \bar{O})}{N\sigma_s\sigma_o}$$

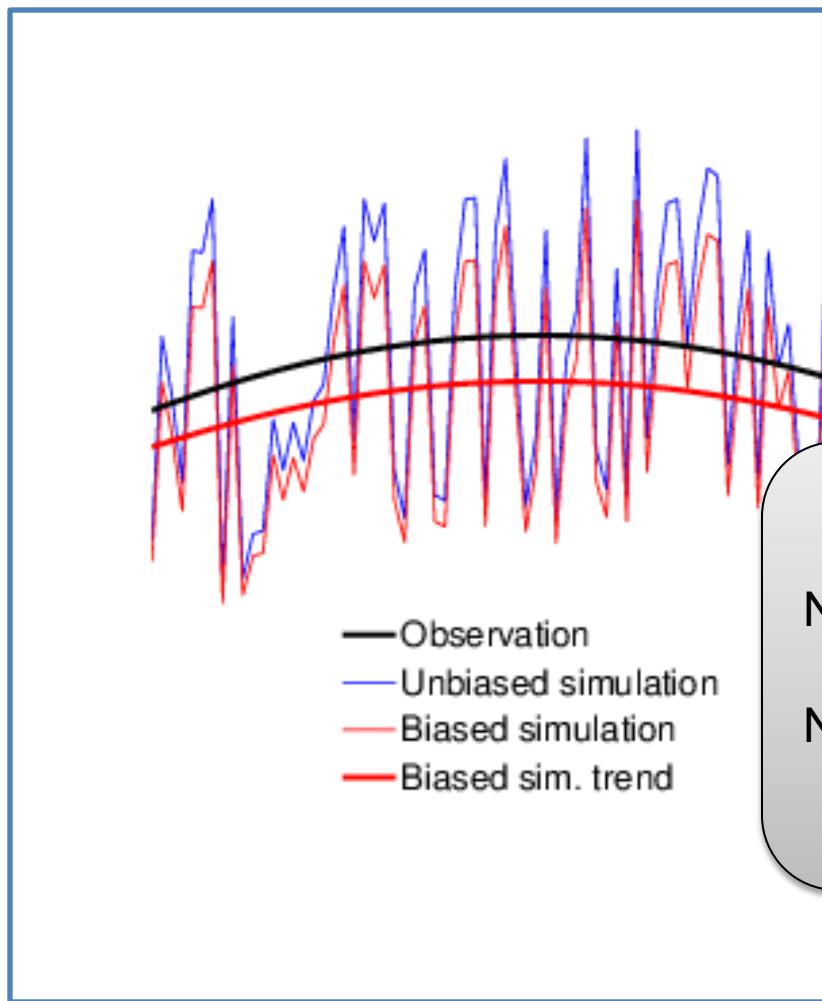
# Statistics on 17 storms and 23 buoys

	ACC350	BJA	T & C
NBI	2.1%	-4.6%	-11.2%
$\rho$	0.889	0.885	0.883
NRMSE	0.2864	0.2800	0.2798 (-2.3%)



# Drawback of using RMSE

## A numerical example



$p=0.614$  for both of the simulations

NBI = -12% for the red simulation

**One would say the best simulation is the blue one.**

$\text{NRMSE}(\text{blue}) = 0.384$

$\text{NRMSE}(\text{red}) = 0.356 (\sim -7.2\%)$

# Geometrical decomposition of RMSE

**Scatter component**

$$SC = \sqrt{\sum [(S_i - \bar{S}) - (O_i - \bar{O})]^2}$$

**Bias component**       $BI = \bar{S} - \bar{O}$

$$RMSE^2 = SC^2 + BI^2$$

# Geometrical decomposition of NRMSE

**Scatter component (also called Scatter Index)**

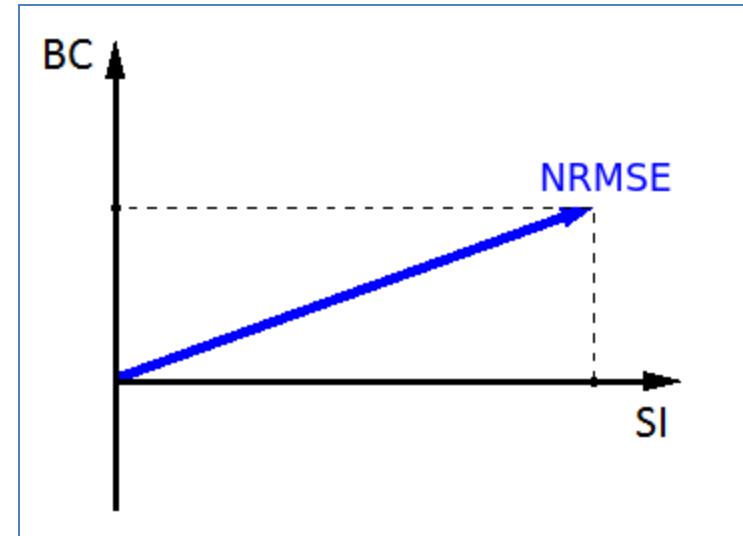
$$SI = \sqrt{\frac{\sum[(S_i - \bar{S}) - (O_i - \bar{O})]^2}{\sum O_i^2}}$$

**Bias component**

$$BC_{NRMSE} = \sqrt{\frac{\bar{O}^2}{\bar{O}^2 + \sigma_o^2}} NBI$$

$$NRMSE^2 = SI^2 + BC_{NRMSE}^2$$

Are SI and BC independent?



In general  $\sigma_s$  and  $\bar{S}$  are not independent.

Let's assume this relation:

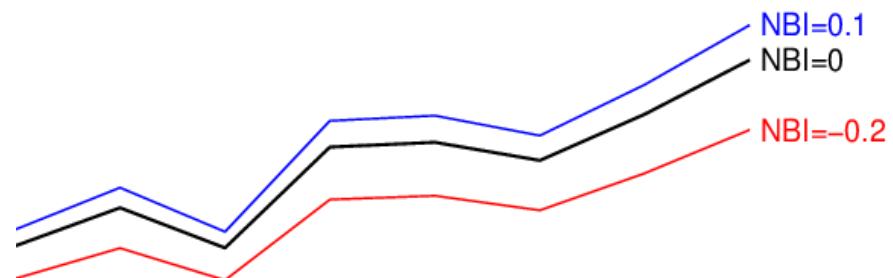
$$\frac{\sigma_s}{\bar{S}} \sim const.$$

Holds for amplifications

Example of set of simulations with constant ratio  $\sigma_s / \bar{S}$  :

$$S_{NBI i} = (1 + NBI) S_{0i}$$

where  $S_{0i}$  is the unbiased simulation.



Amplification factor:

$$\alpha = 1 + NBI$$

$$\frac{\sigma_s}{\bar{S}} \sim const.$$



$$\rho \sim const.$$

# Relationship SI – NBI

We can express both  $\bar{S}$  and  $\sigma_s$  as functions of NBI

- $\bar{S} = \bar{O}(1 + NBI)$
- $\sigma_s / \bar{S} = \text{const.} \Rightarrow \sigma_s = \sigma_{s0}(1 + NBI)$

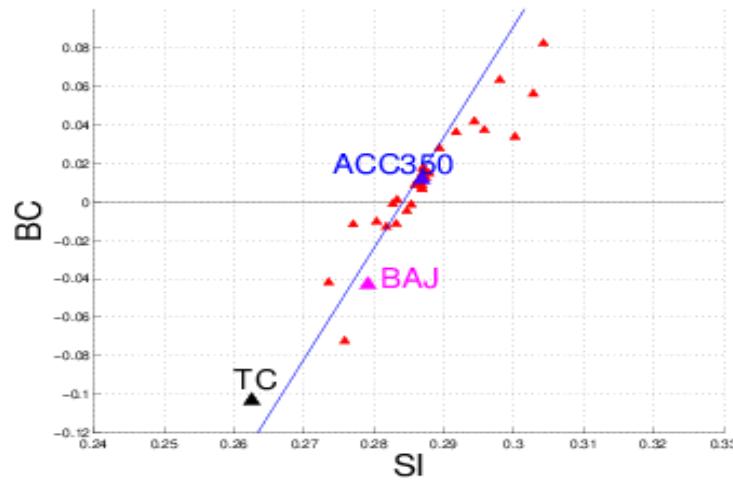
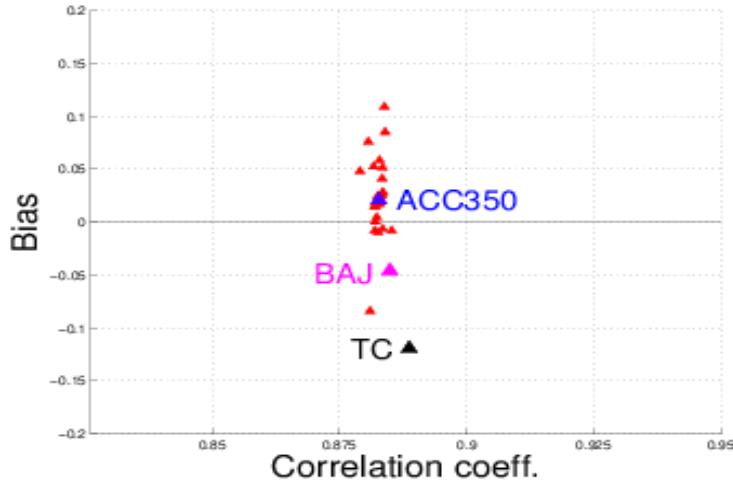
Also SI can be expressed as a function of NBI

$$\sigma_{s0} \sim \sigma_o \quad \xrightarrow{\hspace{1cm}} \quad \boxed{SI \sim SI_0 \left( 1 + \frac{1}{2} \textcolor{red}{NBI} \right)}$$
$$\boxed{\left. \frac{\partial SI}{\partial NBI} \right|_{NBI=0} \sim \frac{1}{2} SI_0 > 0}$$

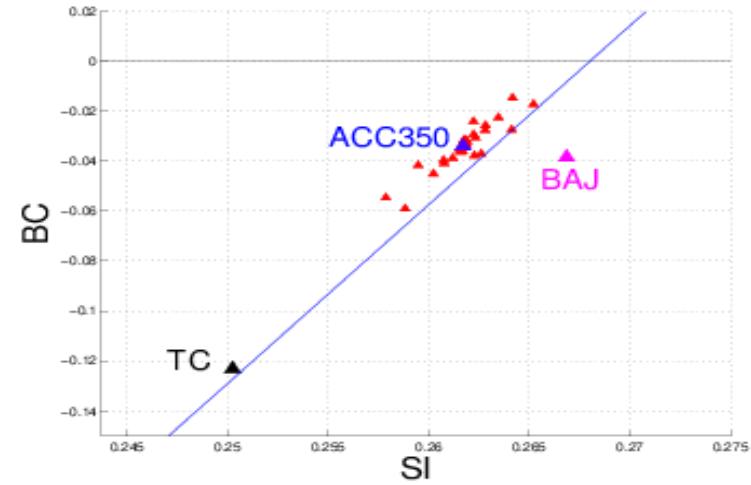
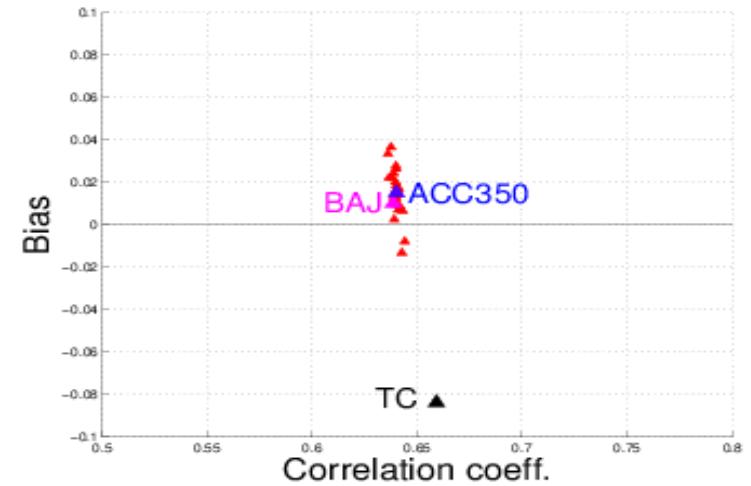
SI grows linearly in NBI around NBI=0

$$SI \sim SI_0 \left( 1 + \frac{1}{2} NBI \right)$$

Fits well real world data



Significant wave height

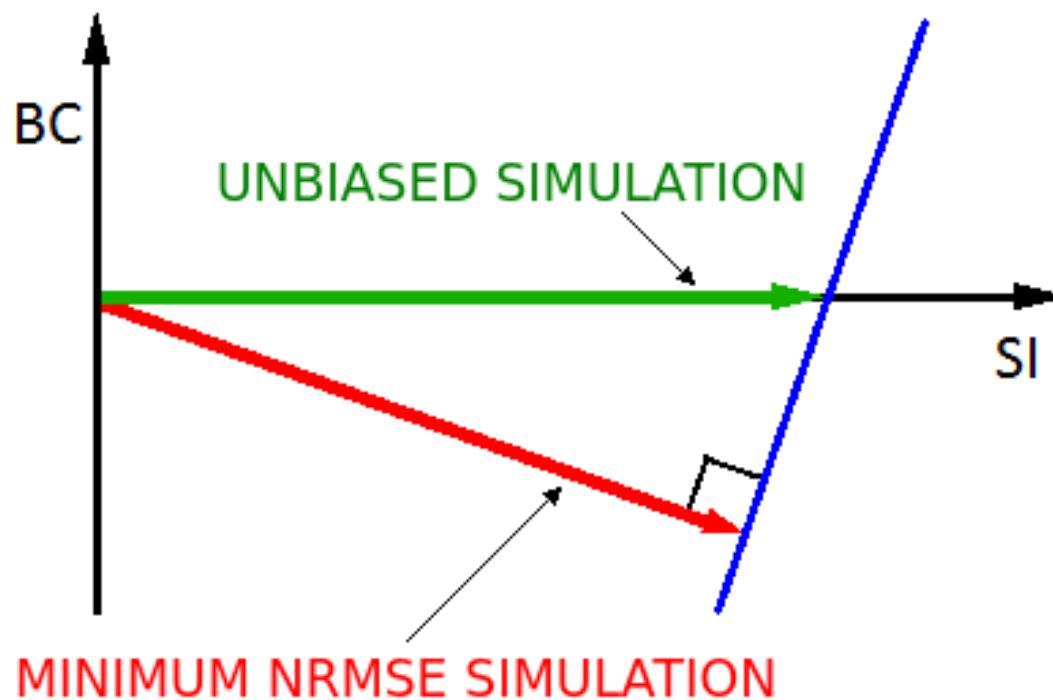


Mean period

$$SI \sim SI_0 \left( 1 + \frac{1}{2} NBI \right)$$



The simulation with the minimum value of RMSE/NRMSE underestimates the average value



# Conditions when this effect is most evident

- Standard deviation of the same order of the mean

$$\frac{\sigma_o}{O} \approx 1$$

- Correlation appreciably smaller than 1 (0.7 - 0.9), since SI is minimum for

$$NBI = 1 - \rho$$

# How to overcome this problem?

Hanna and Heinold (1985) indicator:

$$HH = \sqrt{\frac{\sum (S_i - O_i)^2}{\sum S_i O_i}} = \sqrt{\frac{(S - O)^2}{SO}}$$

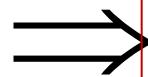
Property of HH:  
ρ constant  HH minimum for null bias

$$HH^2 = \frac{\sum (S_i - O_i)^2}{\sum S_i O_i} = \frac{\bar{S}^2 + \sigma_s^2 + \bar{O}^2 + \sigma_o^2}{\bar{S}\bar{O} + \rho\sigma_s\sigma_o} - 2$$

- $\bar{S} = \bar{O}(1 + NBI)$
- $\sigma_s / \bar{S} = \text{const.} \Rightarrow \sigma_s = \sigma_{s0}(1 + NBI)$
- $\sigma_{s0} \sim \sigma_o$

$$\left. \frac{\partial HH^2}{\partial NBI} \right|_{NBI=0} \sim 0$$

$$\left. \frac{\partial^2 HH^2}{\partial NBI^2} \right|_{NBI=0} \sim 2 \frac{\bar{O}^2 + \sigma_o^2}{\bar{O}^2 + \rho\sigma_o^2} > 0$$



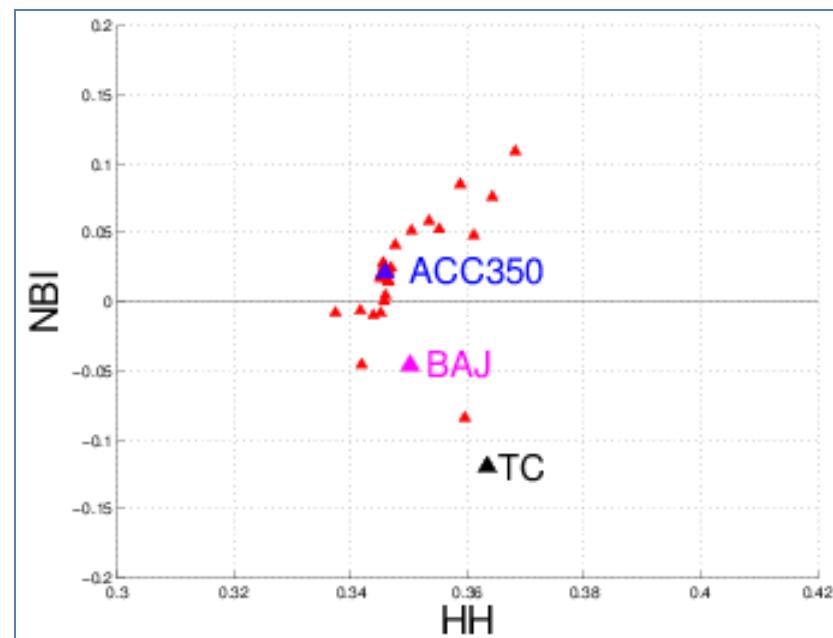
HH has a minimum  
for null bias

# Wavewatch III validation on the Mediterranean sea.

	ACC350	BJA	T & C
NBI	2.1%	-4.6%	-11.2%
$\rho$	0.889	0.885	0.883
NRMSE	0.2864	0.2800	0.2798 (-2.3%)
HH	0.3459	0.3502	0.3634 (+4.8%)

HH has a minimum  
for null bias

Mentaschi et al. 2013



# Conclusions

- RMSE, NRMSE and SI tend to be **systematically better for simulations affected by negative bias**. This is mostly evident when:
  - We are tuning parameters involving an amplification of the results.
  - Correlation coefficient appreciably smaller than 1,  $\rho < 0.9$ .
  - Standard deviation of the observations of the same order of the average,  $\sigma_o / \bar{O} \sim 1$
- HH indicator overcomes this problem introducing a different normalization of the root mean square error.

Thank you!