Incorporating Breaking Wave Predictions in Spectral Wave Forecast Models

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Motivational Aspects

- **wave breaking at sea** - widespread air-sea interfacial process with very significant geophysical and maritime importance
- present spectral wave forecast models **do not provide explicit forecasts** of breaking wave properties.
- recent advances in understanding wave breaking have made it possible to redress this deficiency
- to describe a novel methodology that adds to standard spectral wave model output - **accurate forecasts** of

(i) the **spectral density of breaking crest length per unit area** and

(ii) the associated **breaking strength**

We did this initially for the **dominant wind waves** and have now extended it across the **full spectrum**

Mail Online News

St Jude's Storm shutdown: Eurostar and Monday morning train services cancelled across south as coast is lashed by 25ft waves

- Amendments and cancellations on First Capital Connect, Southeastern, Greater Anglia and Stansted Express
- Also disruption on East Coast, c2c, First Great Western, Southern, Gatwick Express and South West Trains
- Ferries from Poole and Weymouth to Guernsey & Jersey cancelled and hovercrafts to Isle of Wight suspended
- About 60 flights cancelled at London Heathrow Airport tomorrow but none yet at Gatwick, Stansted and Luton
- Forecasters warn houses face damage, trees falling and power cuts in biggest storm to hit Britain in a decade
- Wales and South West England will be hit first early tomorrow morning with winds of up to 90mph expected
- Boy, 14, believed to have drowned today after swimming with friends in waves off Newhaven in East Sussex
- Canoeist dies after being pulled from swollen River Tees near Barnard Castle, County Durham, after capsizing

By MARK DUELL, TARA BRADY and JONATHAN PETRE


WITHIN
200 METRES
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SEALINK
BRITISH FERRIES
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Modeling Wave Breaking Spectrally

Radiative transfer equation (deep water, no currents)

The radiative transfer equation for describing the evolution of the wave height spectrum *F(k)* is given by:

$$\frac{\partial F}{\partial t} + c_g \cdot \nabla F = S_{in} + S_{nl} + S_{diss}$$

where

- *F=F(k,θ)* is the directional wave spectrum
- c_g is the group velocity
- $S_{tot} = S_{in} + S_{nl} + S_{ds}$ is the total source term.
- S_{in} is the ***atmospheric input*** *spectral source term*
- S_{nl} is the ***nonlinear spectral transfer*** *source term representing nonlinear wave-wave interactions within the spectrum*
- S_{ds} is the spectral ***dissipation rate*** *here taken as due primarily to wave breaking*

Saturation Threshold-based Dissipation Rate S_{ds}

- based on treating spectral bands as nonlinear wave groups. Uses a low power of the spectral saturation ratio (~steepness ratio) to simulate observed threshold behaviour *[extension of Alves & Banner (JPO, 2003)]*

$$S_{ds}(k,\theta) = [C_1 * D * (\tilde{\sigma} - \tilde{\sigma}_T)/\tilde{\sigma}_T)^{a_1} + C_2 * D * E_{tot} * k_p{}^2 \;]\,(\sigma/\sigma_m)^{a_2}\;\omega F(k,\theta)$$

'local S_{ds}' **'non-local S_{ds}'**

This formulation uses

- normalized azimuthally-integrated saturation: $k^4 F(\mathbf{k})/\theta(k) = (2\pi)^4 f^5 F(f) \,/\, 2g^2$
- measured threshold of the normalized spectral saturation (Banner et al., JPO, 2002) with a_1=2
- tail exponent a_2 = 4 to match dissipation to input behavior in the spectral tail
- nonlocal dissipation rate component
- coefficient D for the local S_{ds}: non-dimensional and linear in the wind speed to match to the wind input term.
- C_1 and C_2 constants

Modified Jansen Wind Input

$$S_{in}(k,\theta) = \varepsilon\,\beta(k,\theta)\,\omega\,(u_*^{red}(k)\cos\theta / c)^2 * E(k,\theta)$$

$$\beta(k,\theta) = J_2\,\mu(\ln(\mu))^4/\kappa^2$$

where J_2=1.6 (Janssen (1991) used 1.2).

$$\beta(k,\theta) = 0$$ for $$\mu > 1$$

$$\mu(k,\theta) = (u_*/c)^2\,(g z_0/u_*^2)\,\exp(J_1\kappa/(u_*\cos\theta/c)^2)$$

$$z_0 = \frac{0.01 u_*^2}{g} / \sqrt{1 - C_0(\tau_w/\tau)}$$

$$u_*^{red}(k_n) = \sqrt{[\tau_{tot} - J_0\sum_{i=1}^{n}(\tau_w(i) + \tau_{bw}(i))]/\rho_{air}}$$

Brief description of the methodology

$\Lambda(\mathbf{c})$ is the spectral density of breaking wave crest length per unit area

$\Pi(\mathbf{c})$ is the spectral density of the total wave crest length per unit area

The breaking probability $P_{br}(\mathbf{c})$ for wave scales **c** is defined as:

$$\Pr_{br}(\mathbf{c}) = \frac{\int \mathbf{c}\,\Lambda\, d\mathbf{c}}{\int \mathbf{c}\,\Pi\, d\mathbf{c}} \sim \frac{\text{passage rate of breaking wave crests}}{\text{passage rate of wave crests}}$$

Λ(c): spectral density of *breaking wave crest length* per unit area with velocities in the range (c, c+dc) (Phillips,1985)

$$b\frac{\rho}{g}c^5\Lambda(c)dc$$ **wave energy dissipation rate at scale c**

$$b\frac{\rho}{g}c^4\Lambda(c)dc$$ **momentum flux from waves of scale c to currents**

The sea state threshold variable used for breaking probability was the normalised spectral saturation

$$\tilde{\sigma}(k) = \sigma(k) / < \theta(k) >$$

where $\sigma(k)$ is the azimuth-integrated spectral saturation given by

$$\sigma(k) = k^4 \Phi(k)$$

$$= (2\pi)^4 f^5 G(f)/2g^2$$

and $<\theta(k)>$ is the mean spectral spreading width given by

$$< \theta(k) > = \int_{-\pi}^{\pi} (\theta - \bar{\theta}) F(k,\theta) k \, d\theta \Big/ \int_{-\pi}^{\pi} F(k,\theta) k \, d\theta$$

where $\bar{\theta}$ is the mean wave direction, and $\Phi(k)$, $G(f)$ and $F(k,\theta)$ are, respectively, the spectra of wave height as a function of scalar wavenumber, frequency and vector wavenumber.

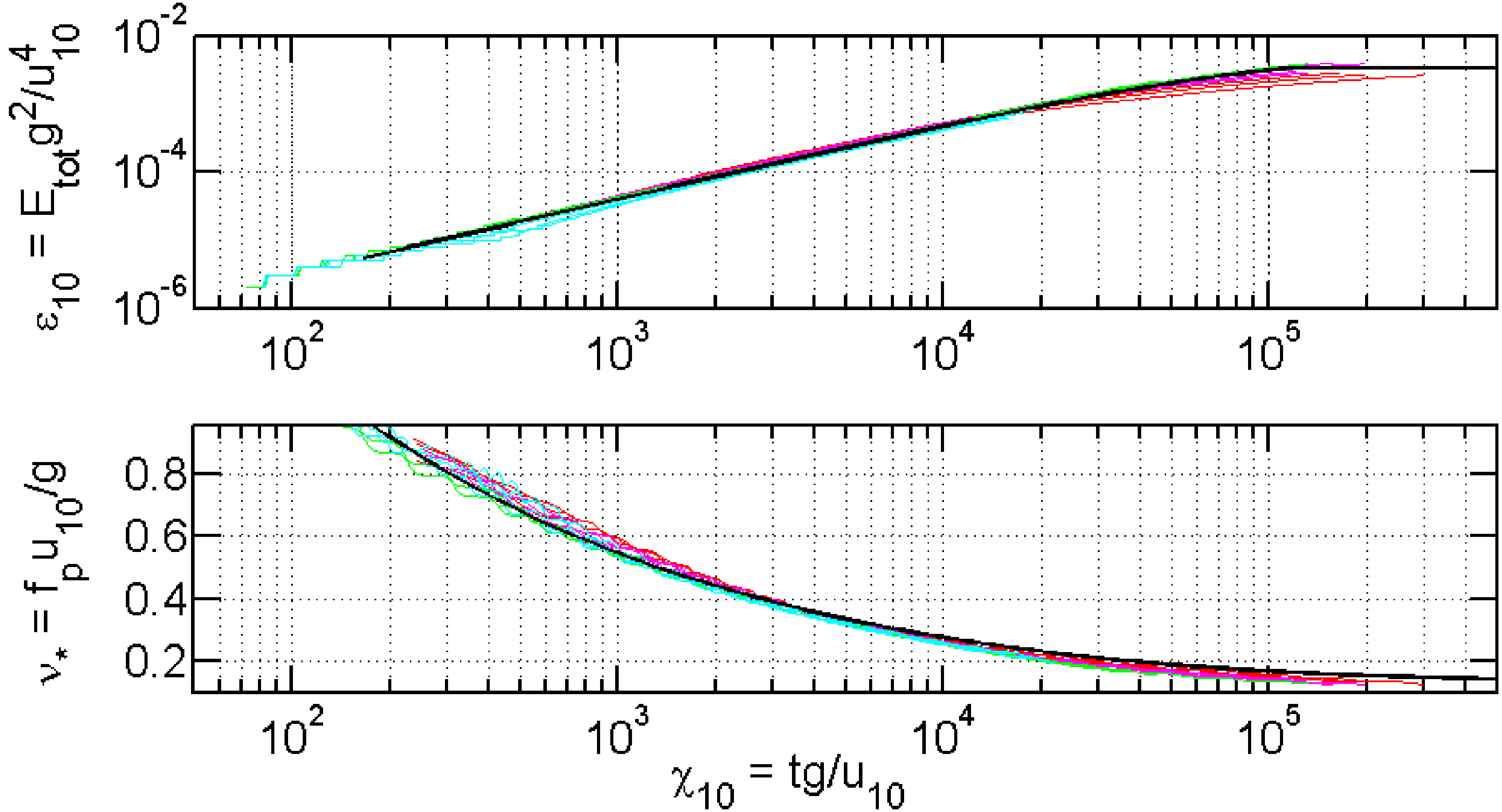
$$\Pr_{br}(\tilde{\sigma}) = H(\tilde{\sigma} - \tilde{\sigma}_T) * \alpha_{br} * \left(\tilde{\sigma} - \tilde{\sigma}_T\right)^{0.5}$$

$$b_{br}(\tilde{\sigma}) = H(\tilde{\sigma} - \tilde{\sigma}_T) * c_{br} * \left(\tilde{\sigma} - \tilde{\sigma}_T\right)^{1.0}$$

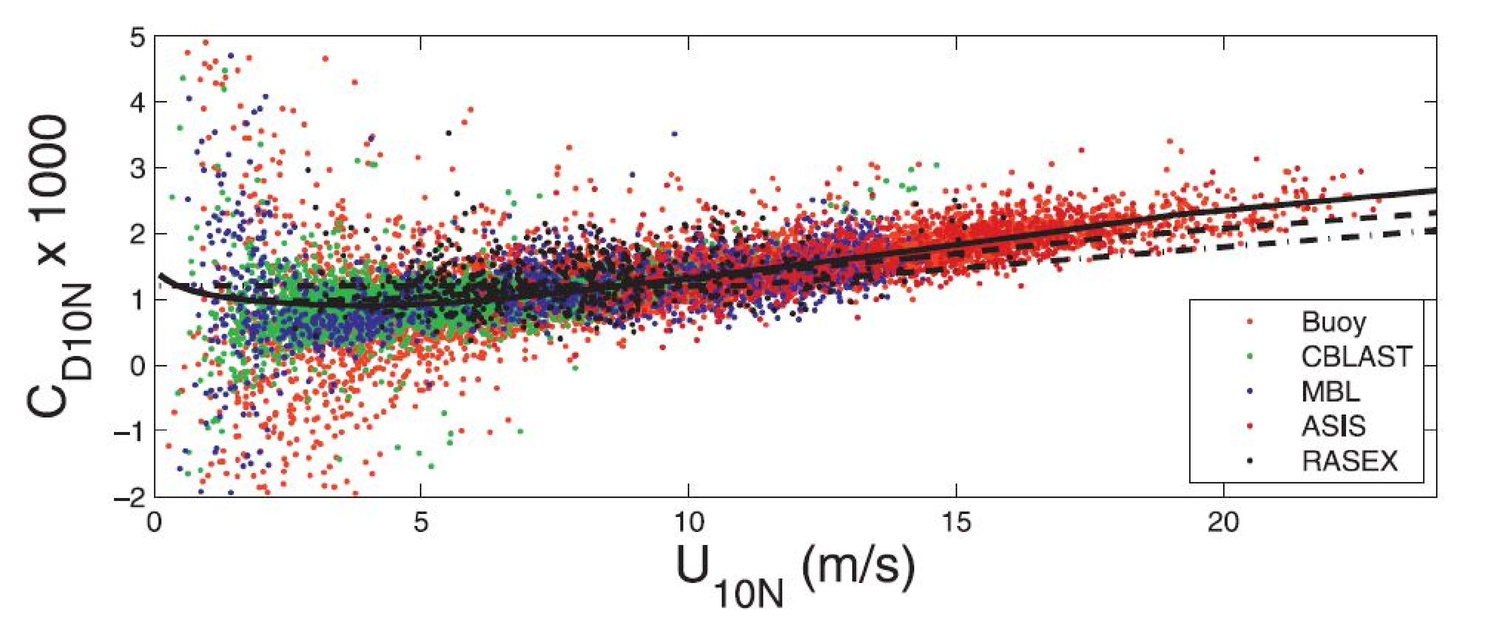
$$S_{ds}^{loc}(c)\, dc = b\, g^{-2} c^5 \Lambda(c)\, dc$$

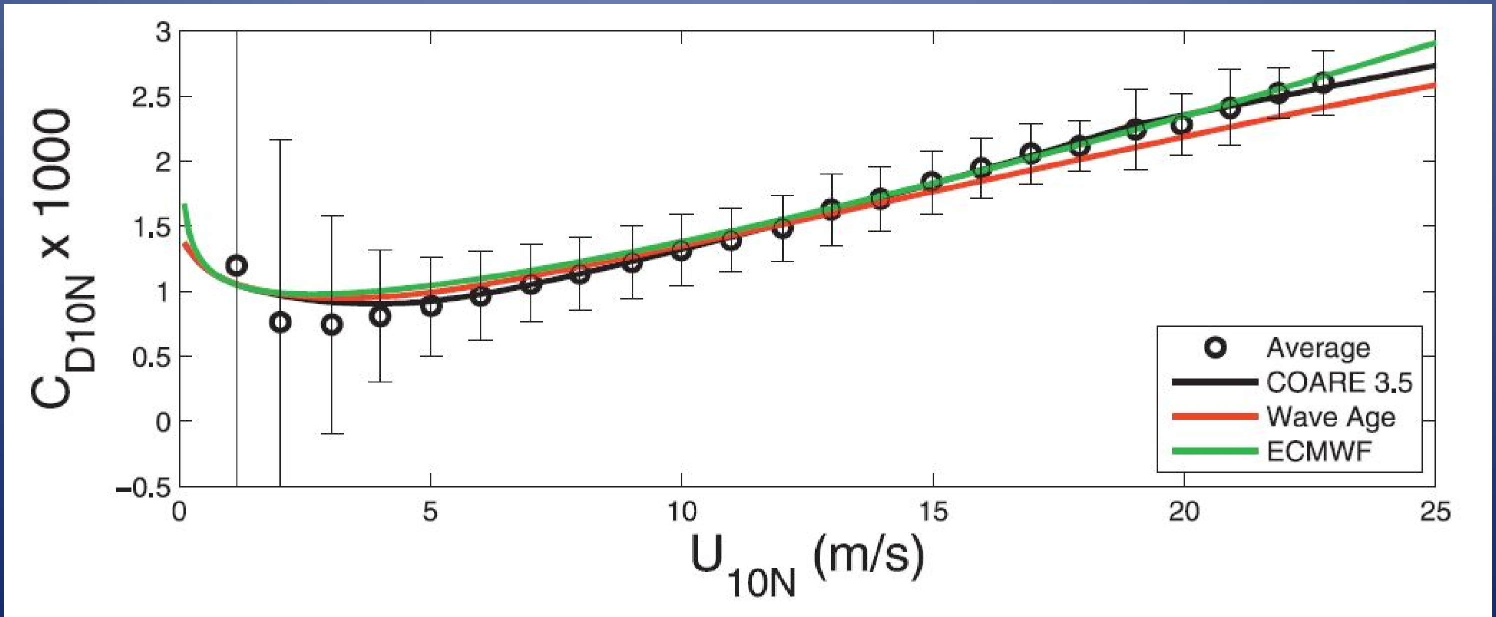
$$\Lambda(c) = S_{ds}^{loc}(c) * g^2/(b * c^5)$$

Non-Dimensional Evolution

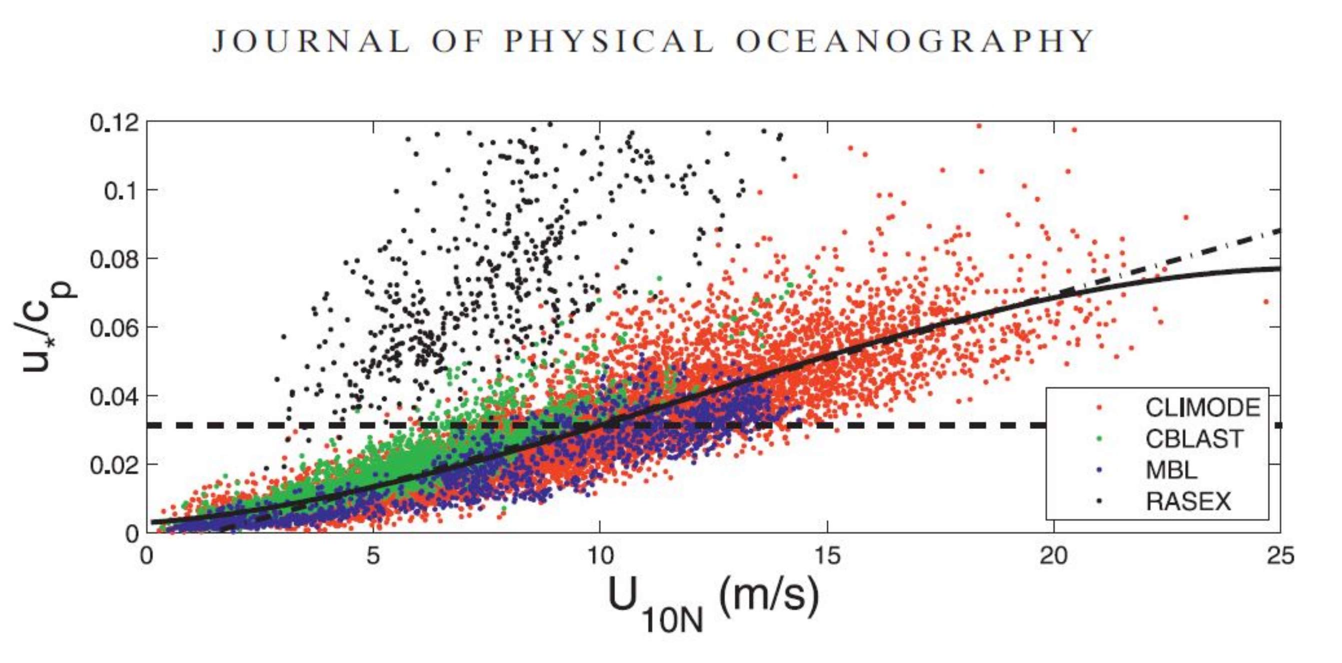


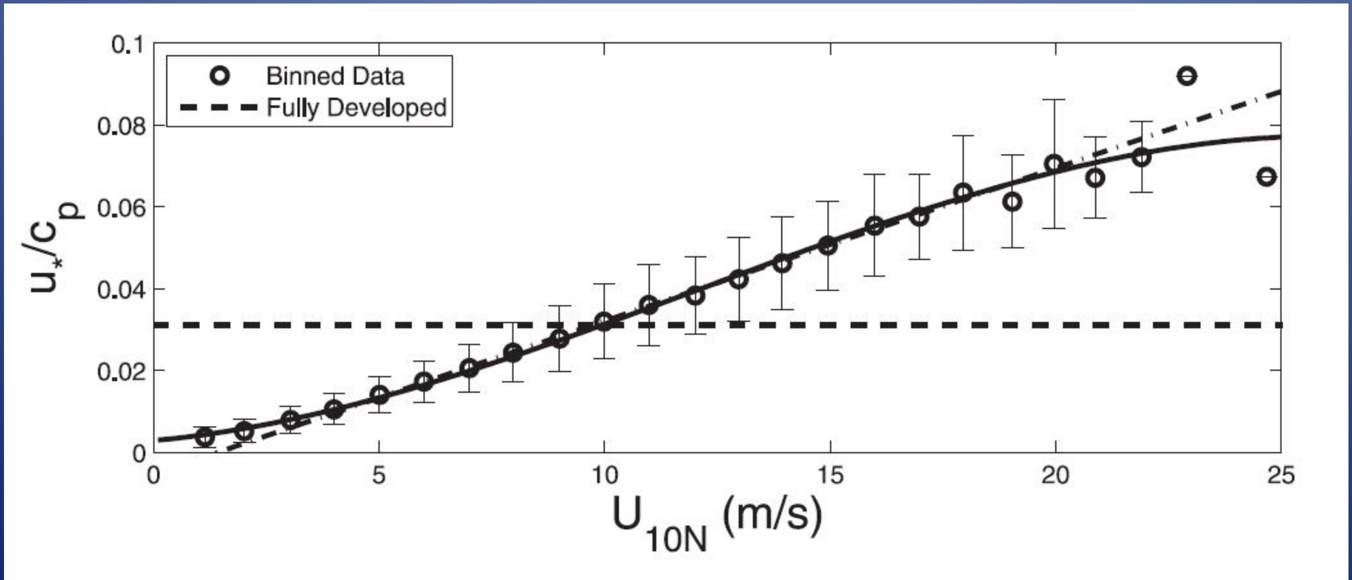
Edson et al, 2013. Drag Coefficient Observational Data.



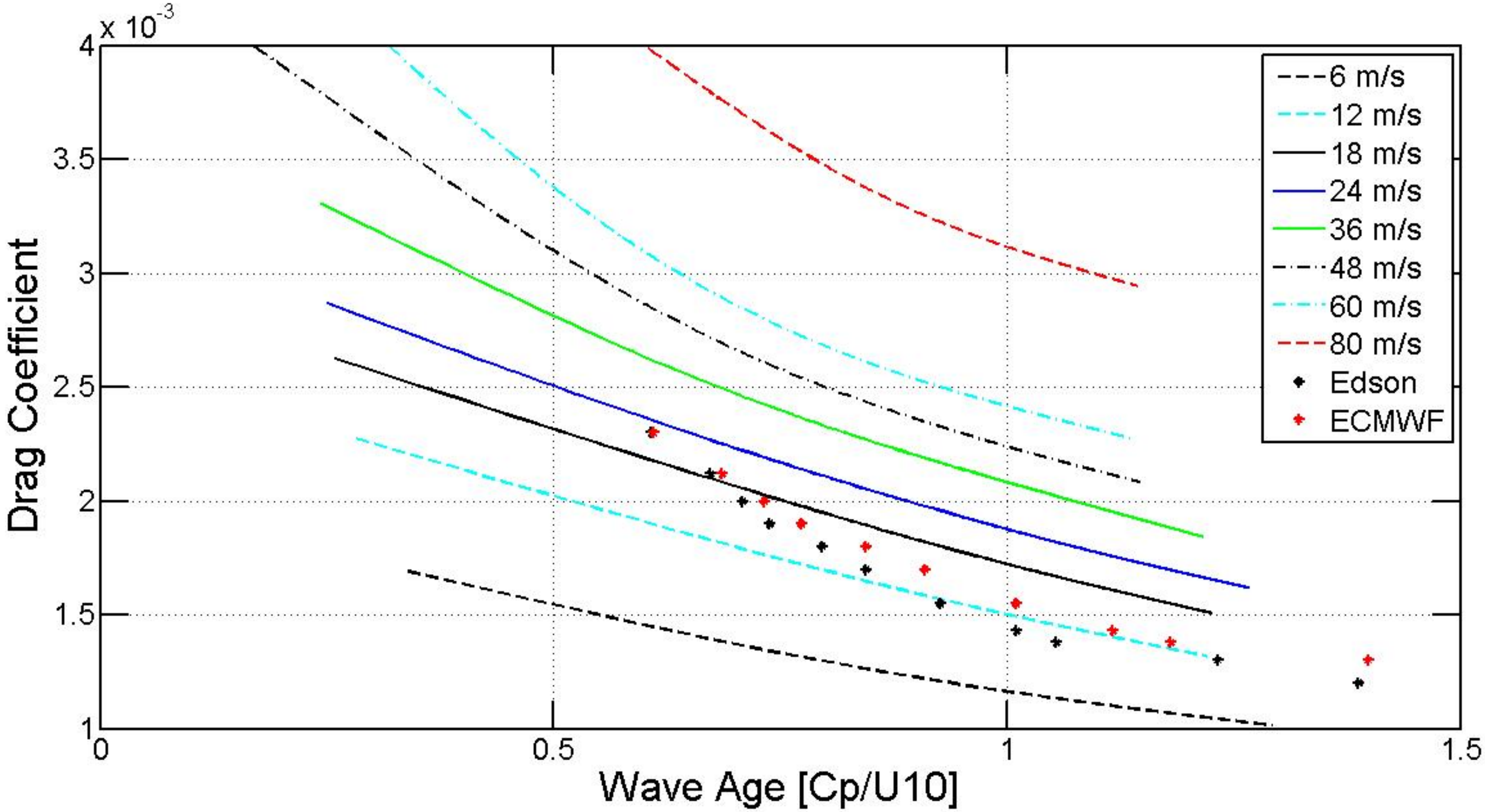


Edson et al, 2013. Observational Data: Wave Age .vs. U10

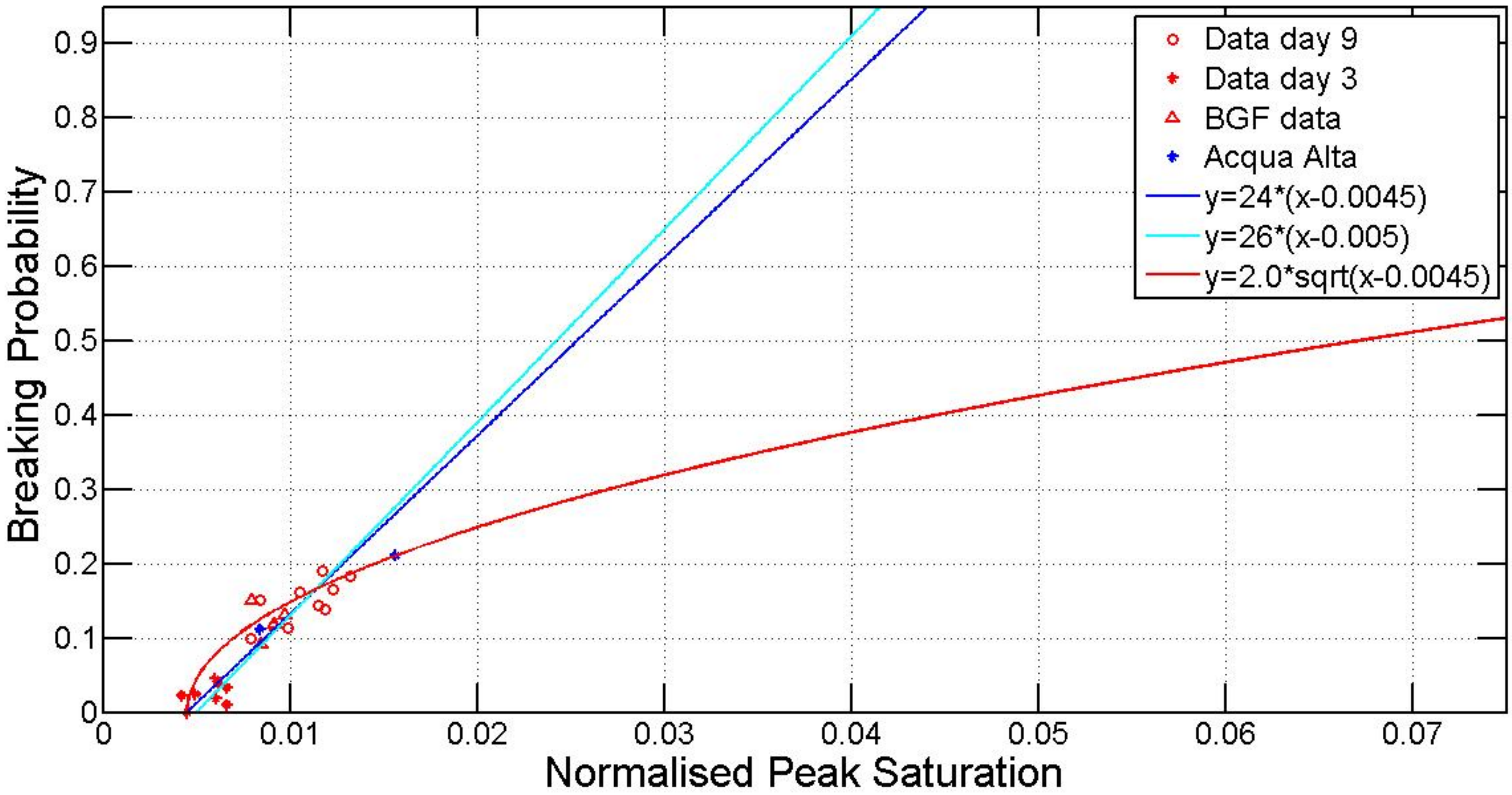




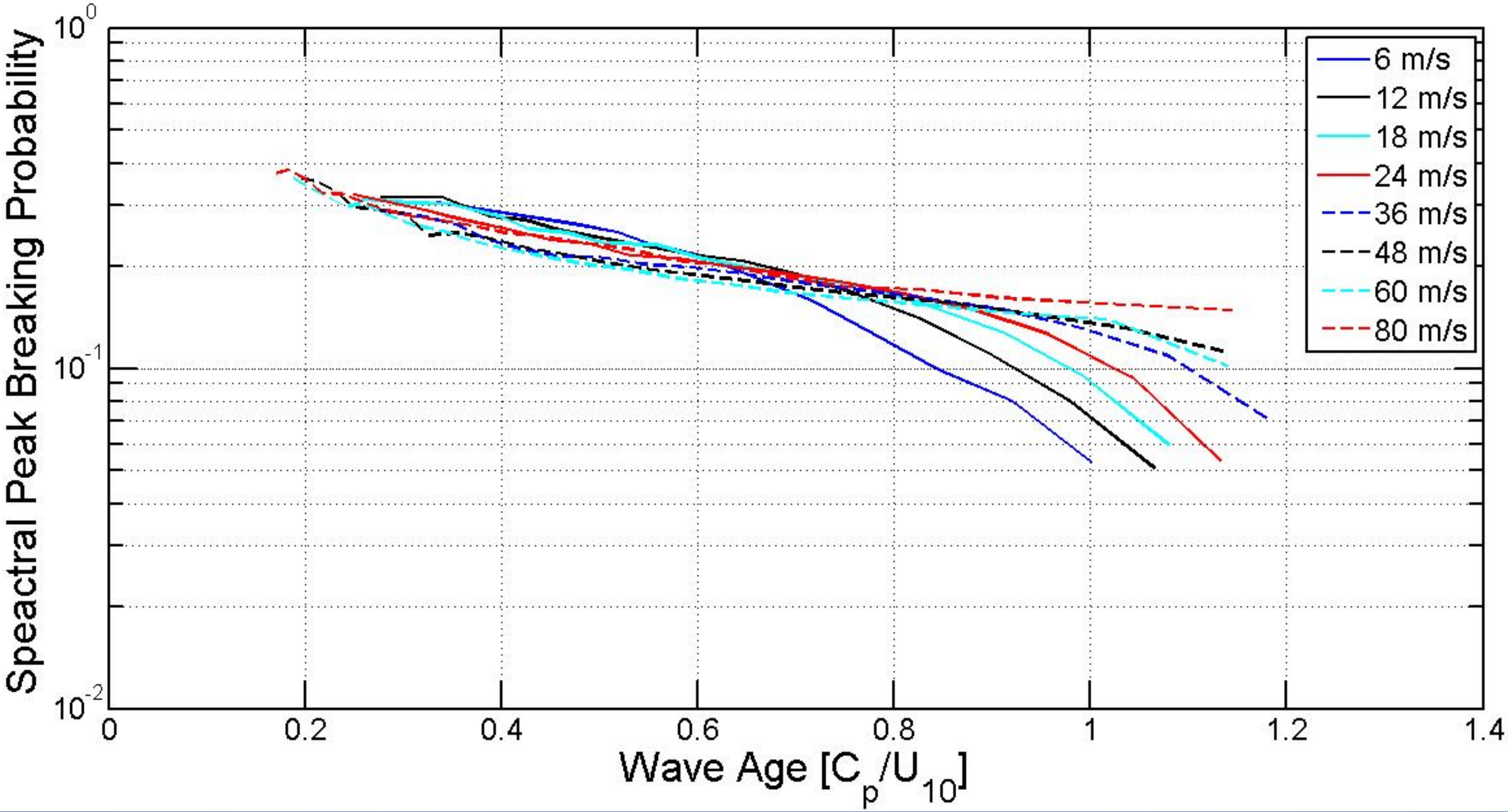
Model Forecast : Drag Coefficient forecast for Fetch Limited for multiple Wind Speeds 6 m/s to 80 m/s



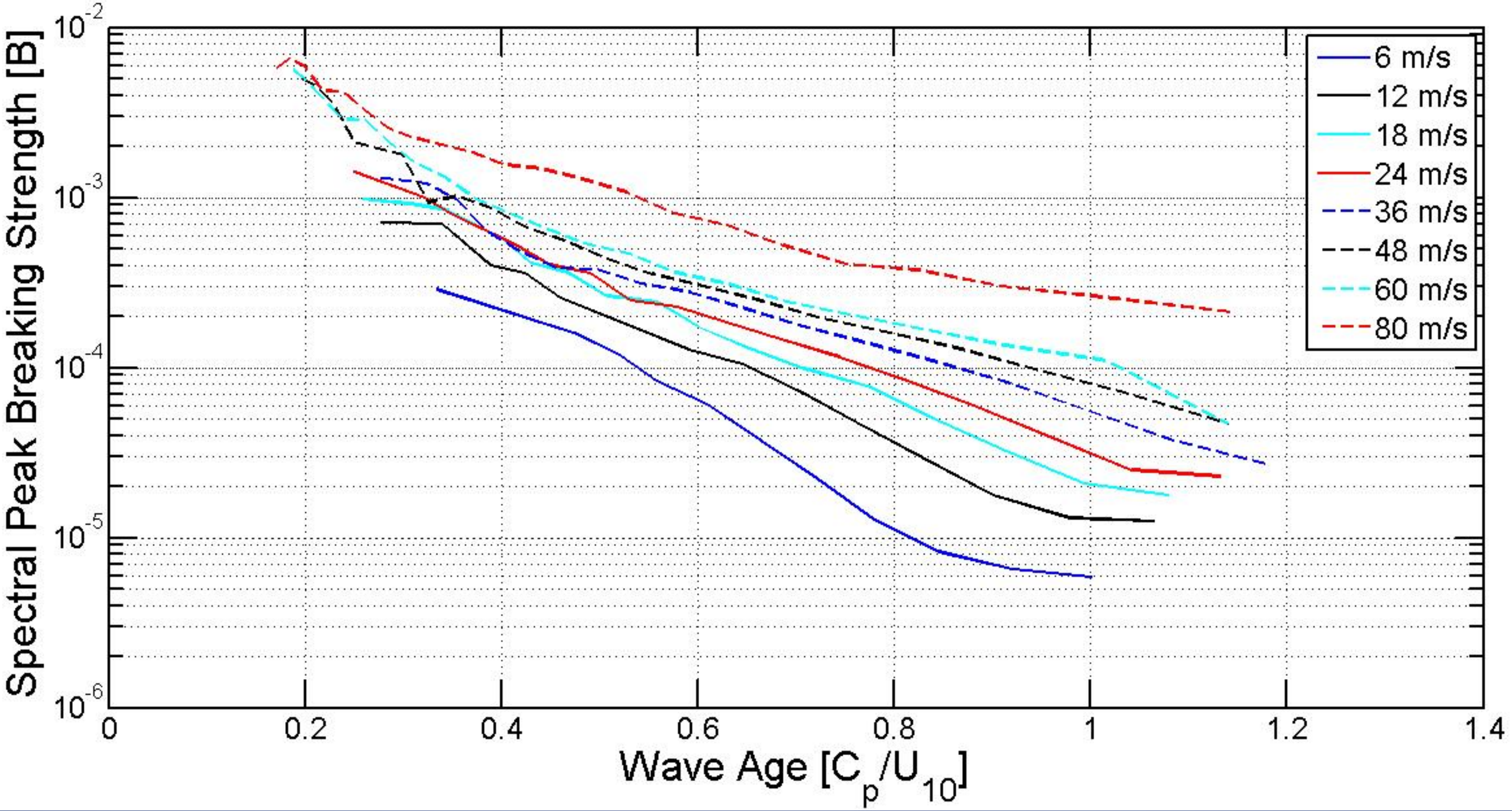
Breaking Probability .vs. Normalized Saturation.
Observed Data; Previous Parameterization; Current Parameterization



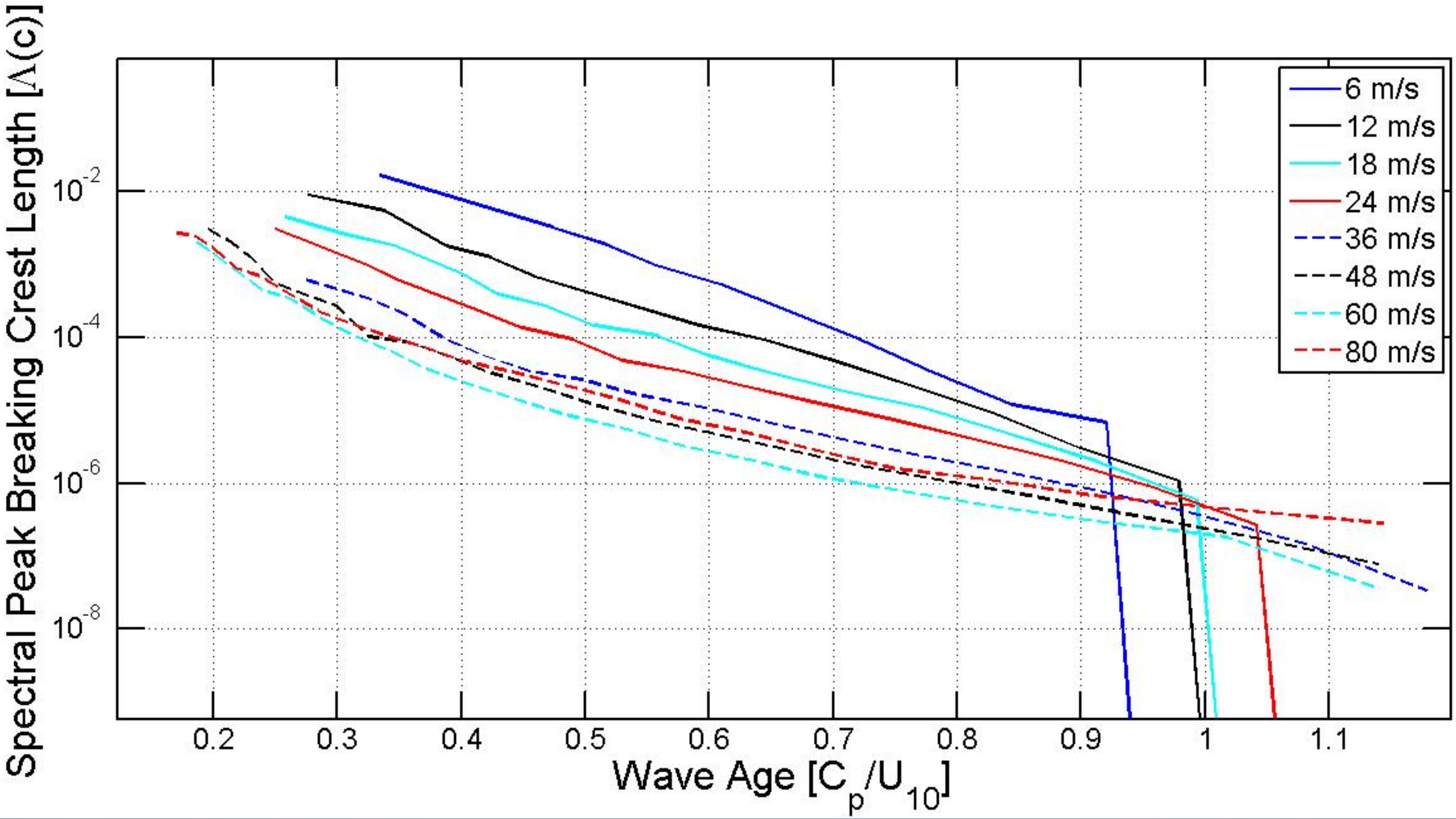
Model Forecast: Breaking Probability .vs. Wave Age for multiple wind speeds 6 m/s to 80 m/s.



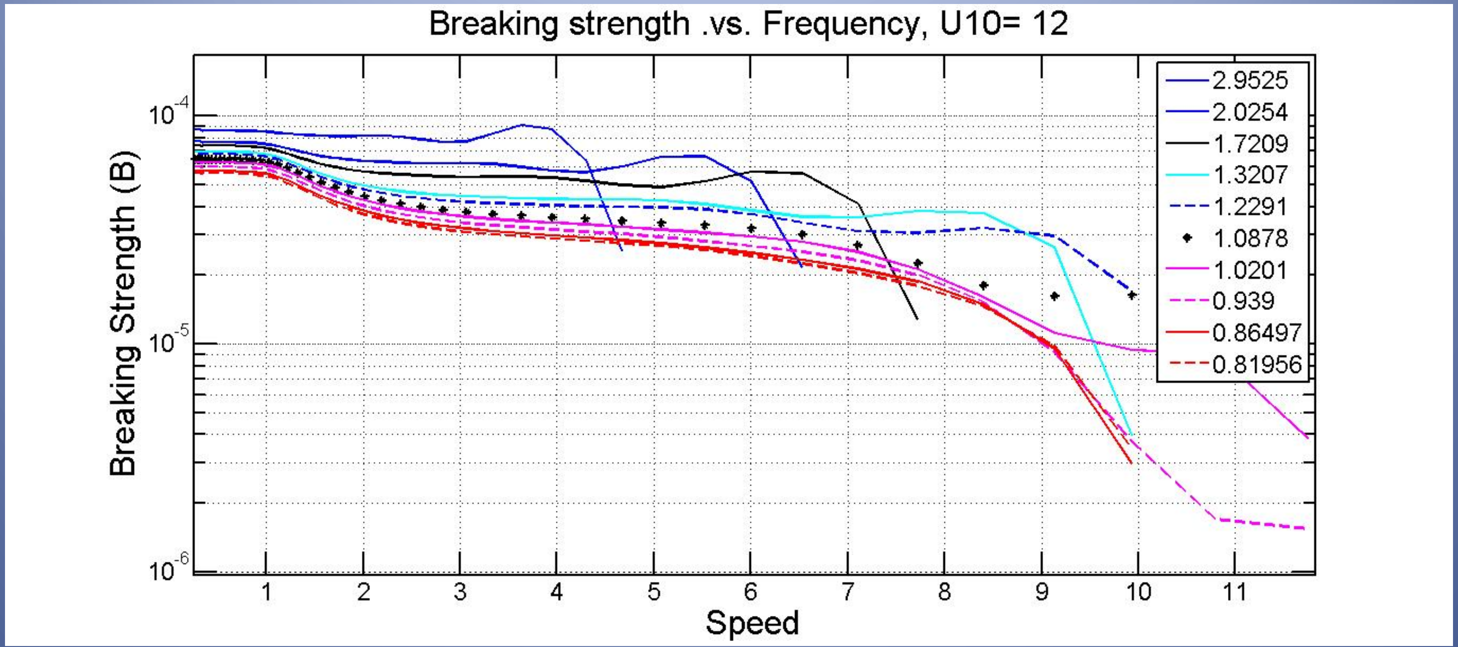
Model Forecast: Breaking Strength .vs. Wave Age For multiple wind speeds 6 ms/ to 80 m/s.

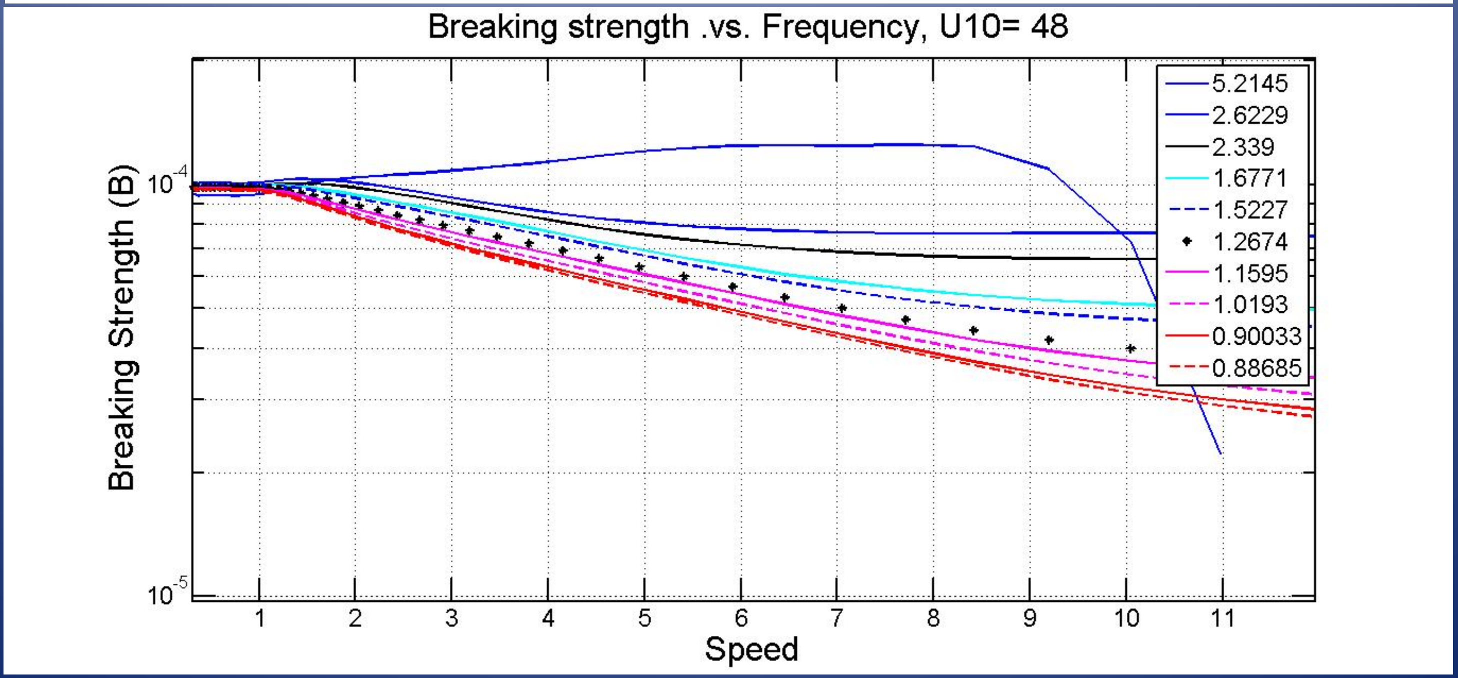


Model Forecast: Lambda(c) .vs. Wave Age.
For multiple wind speeds 6 ms/ to 80 m/s.

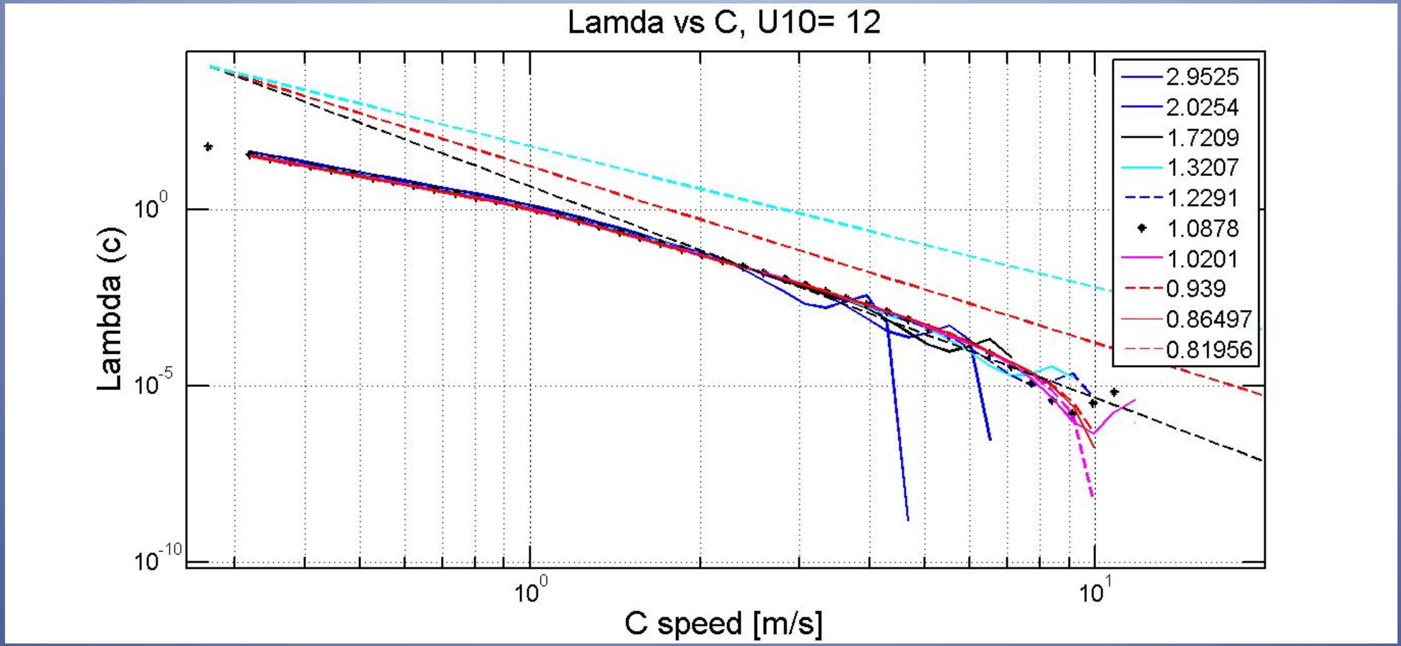


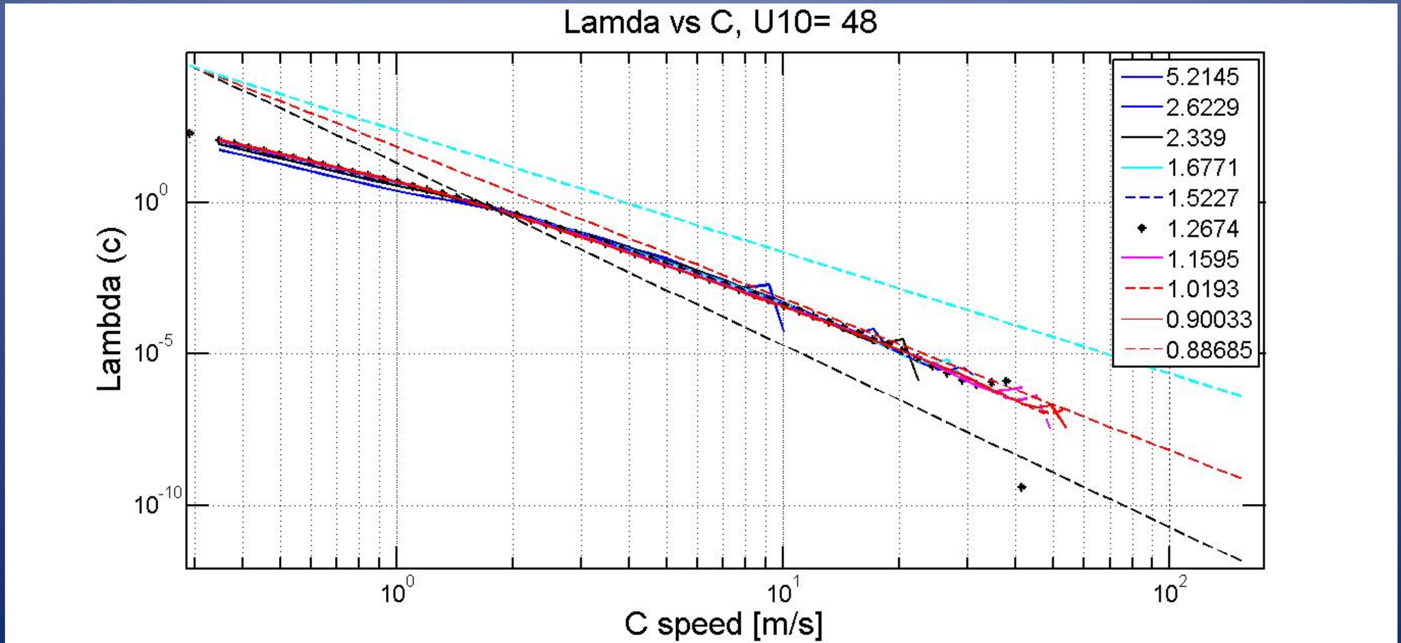
Model Forecast: Breaking Strength [b] .vs. Speed for 12m/s & 48 m/s.

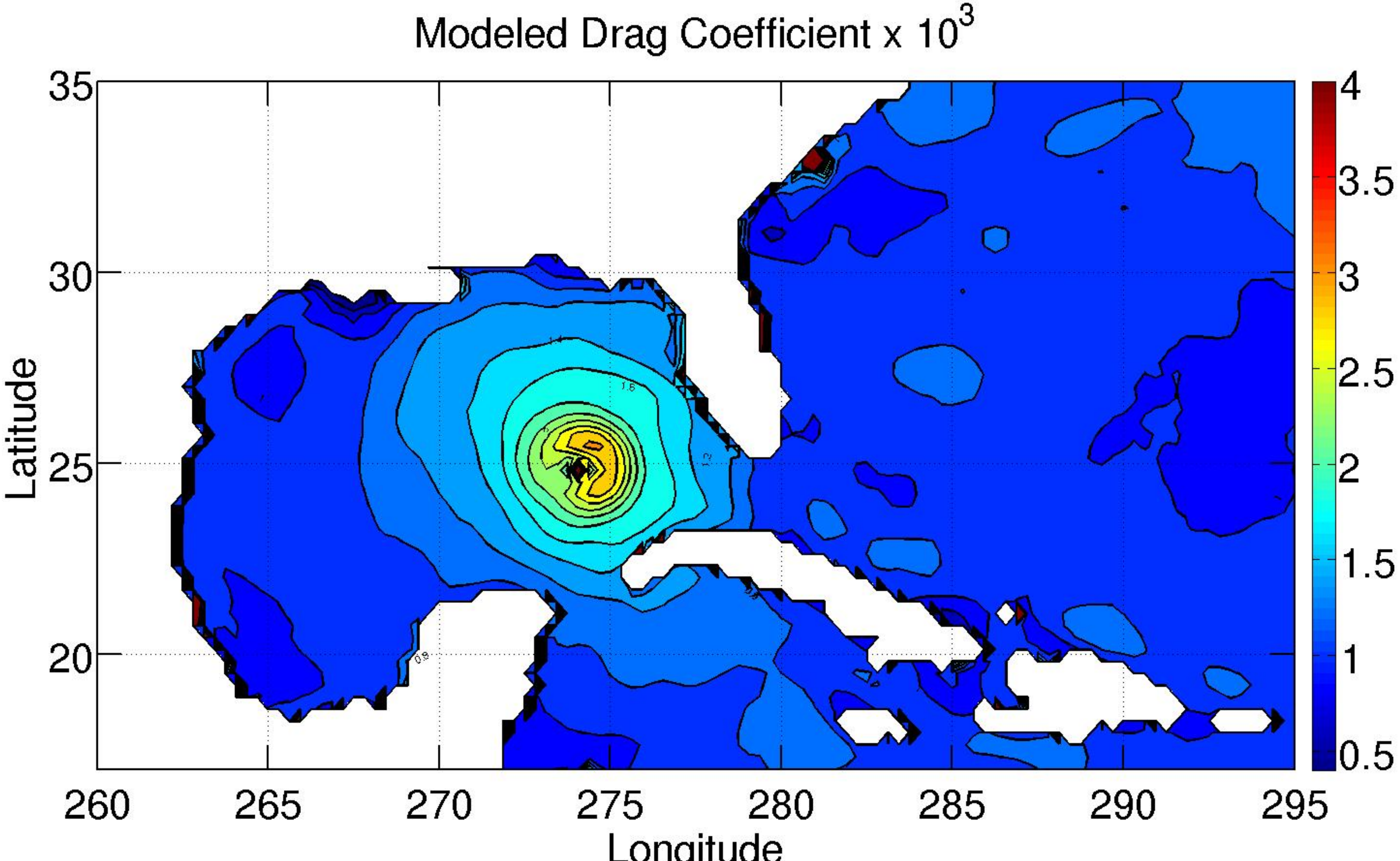




Model Forecast: Lambda(c) .vs. Speed for 12 m/s & 48m/s.

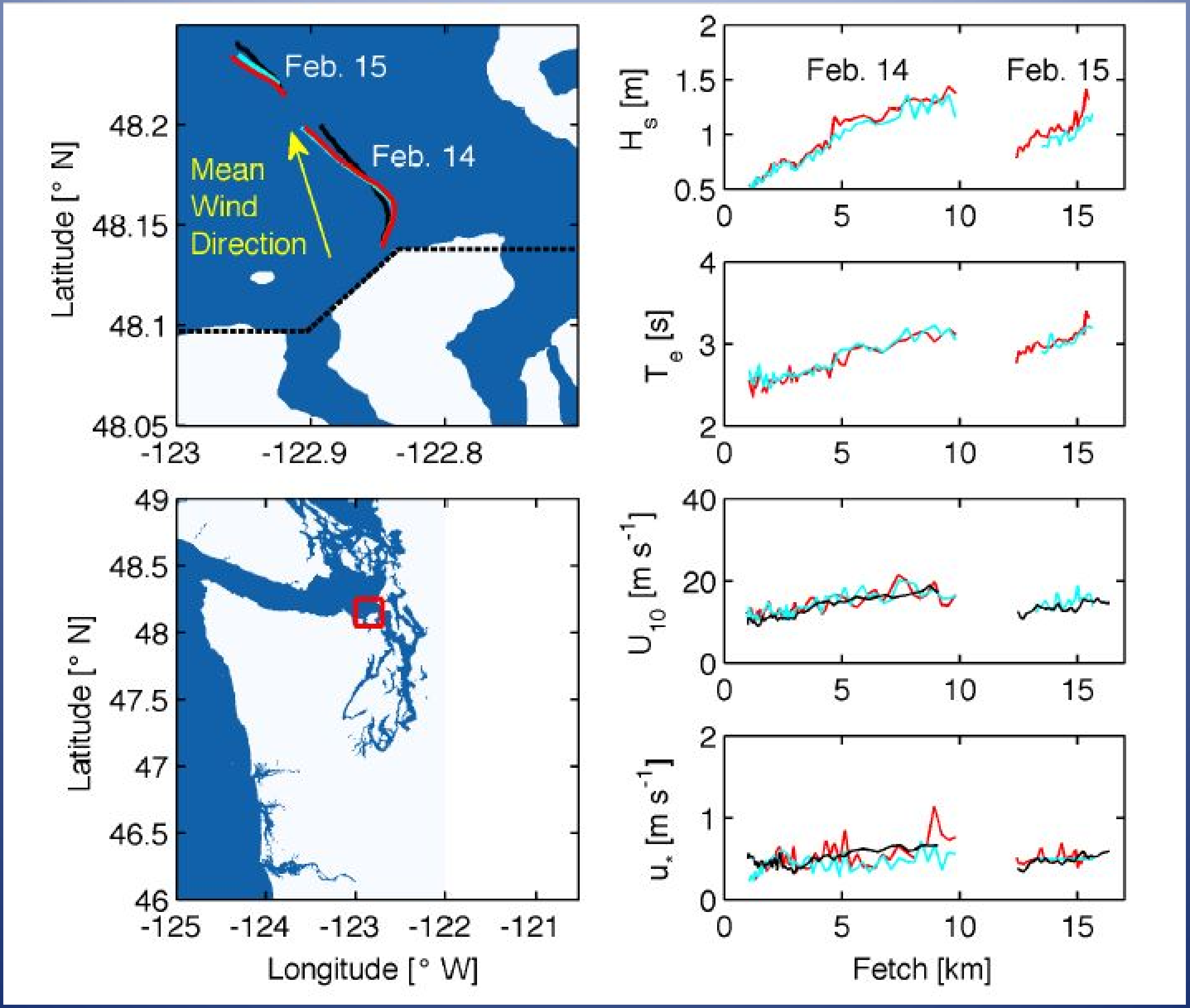


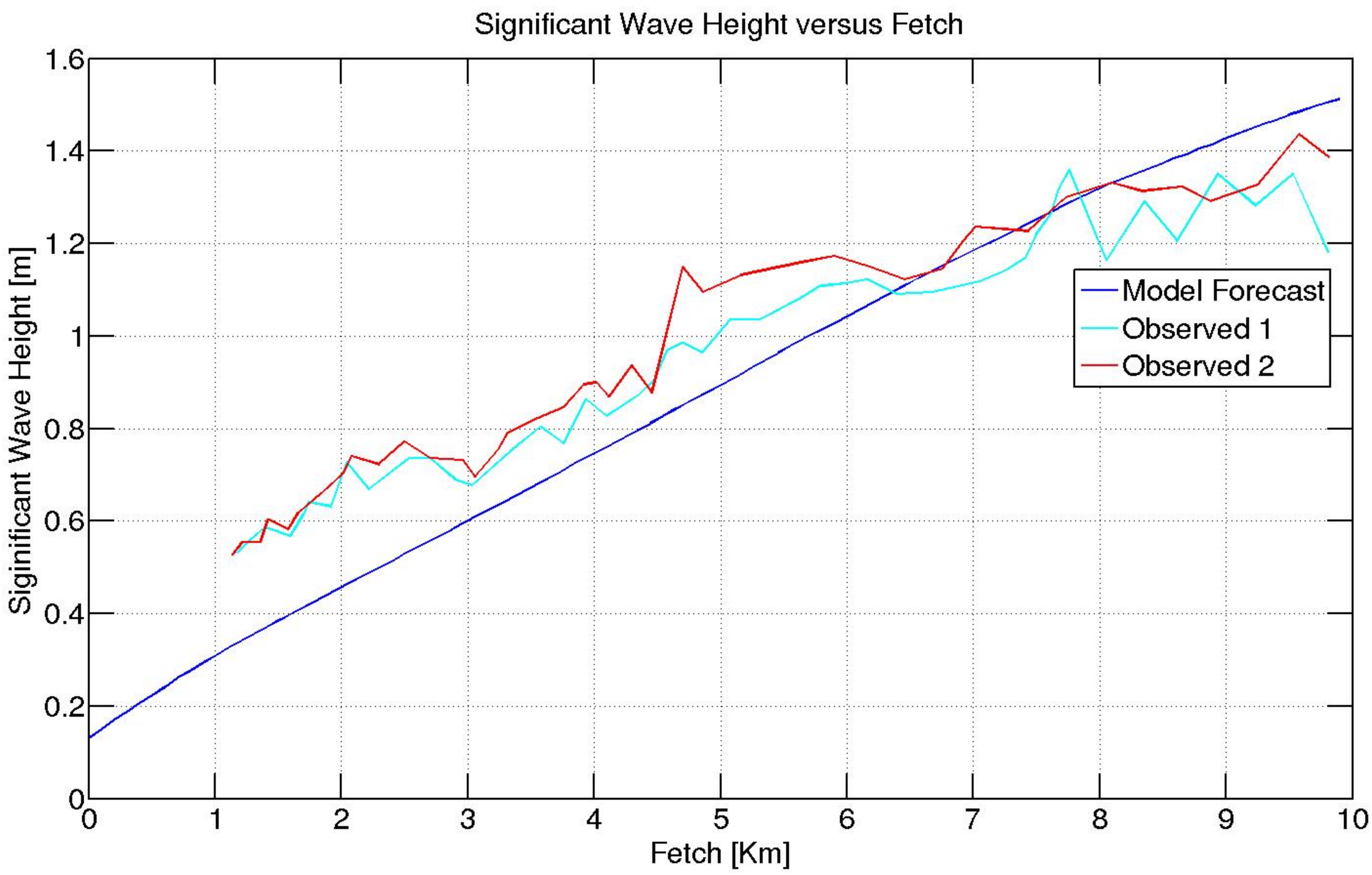


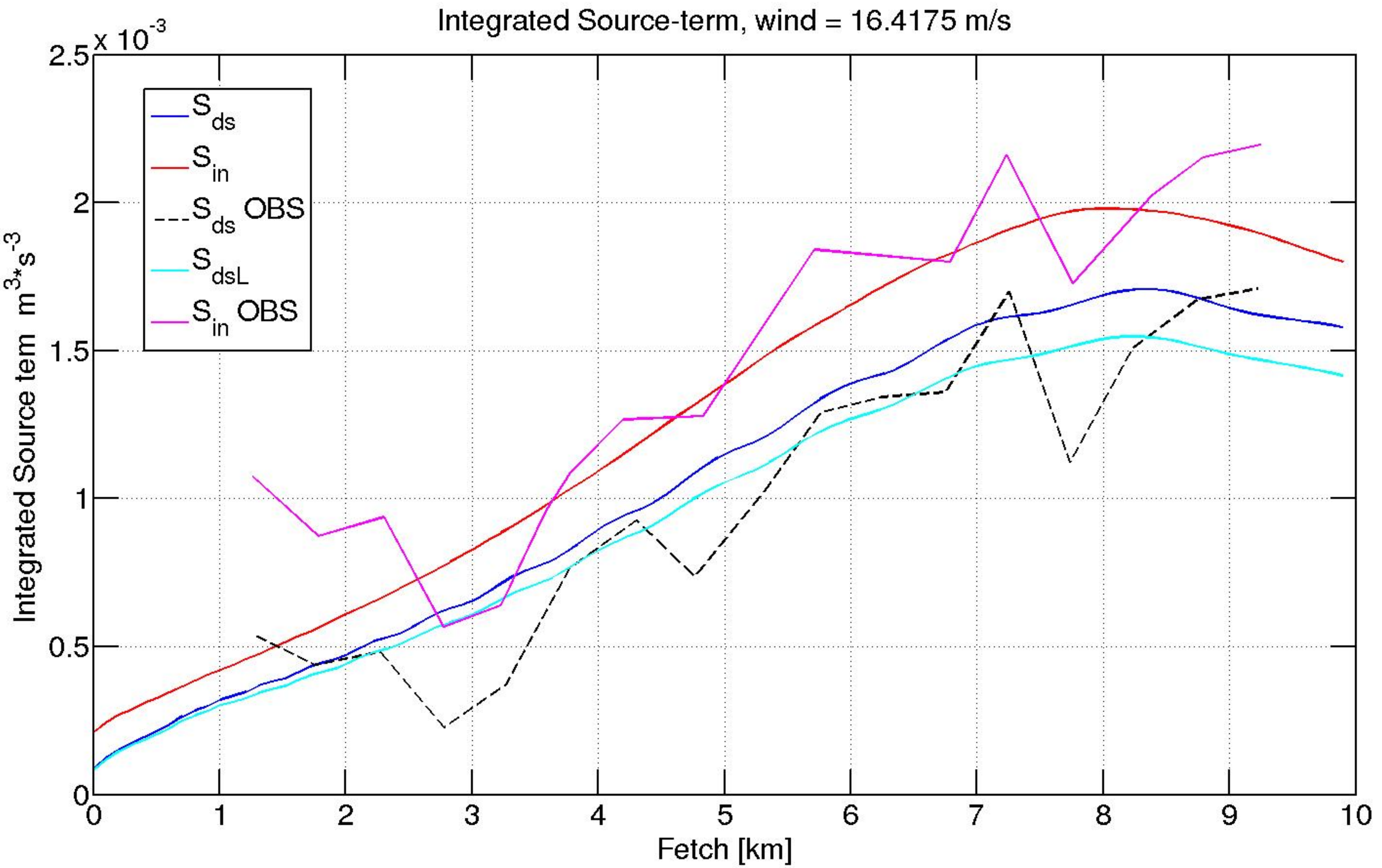


Modeled Drag Coefficient x 10^3
Latitude
Longitude
35
30
25
20
260
265
270
275
280
285
290
295
4
3.5
3
2.5
2
1.5
1
0.5

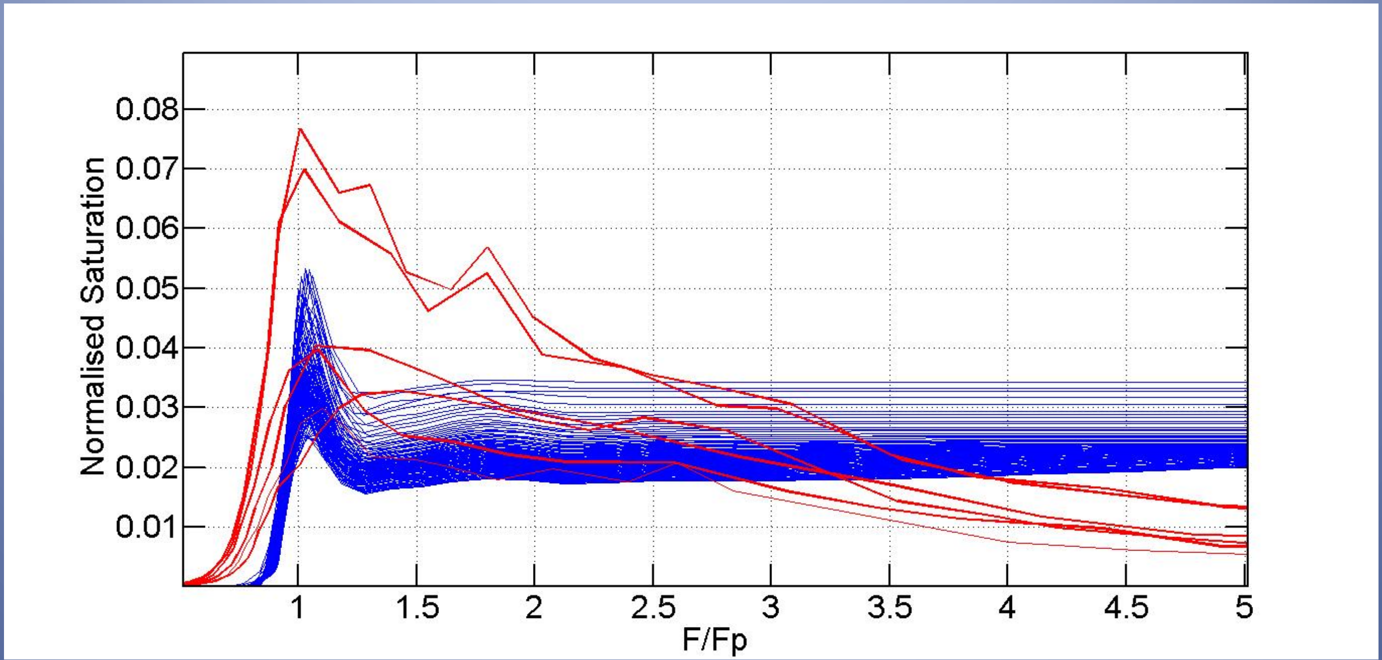
Strait of Juan de Fuca: *Experiment Conditions*



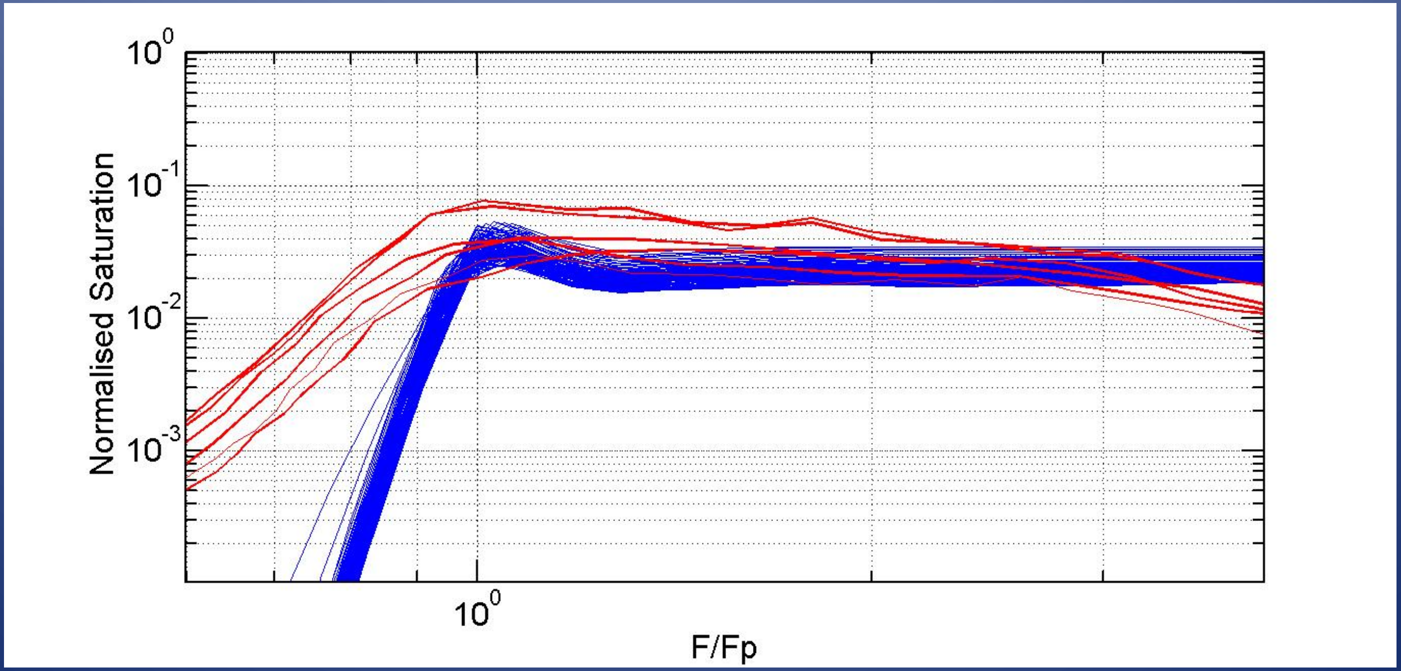

Significant Wave Height versus Fetch
Siginificant Wave Height [m]
Fetch [Km]
Model Forecast
Observed 1
Observed 2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
0
1
2
3
4
5
6
7
8
9
10



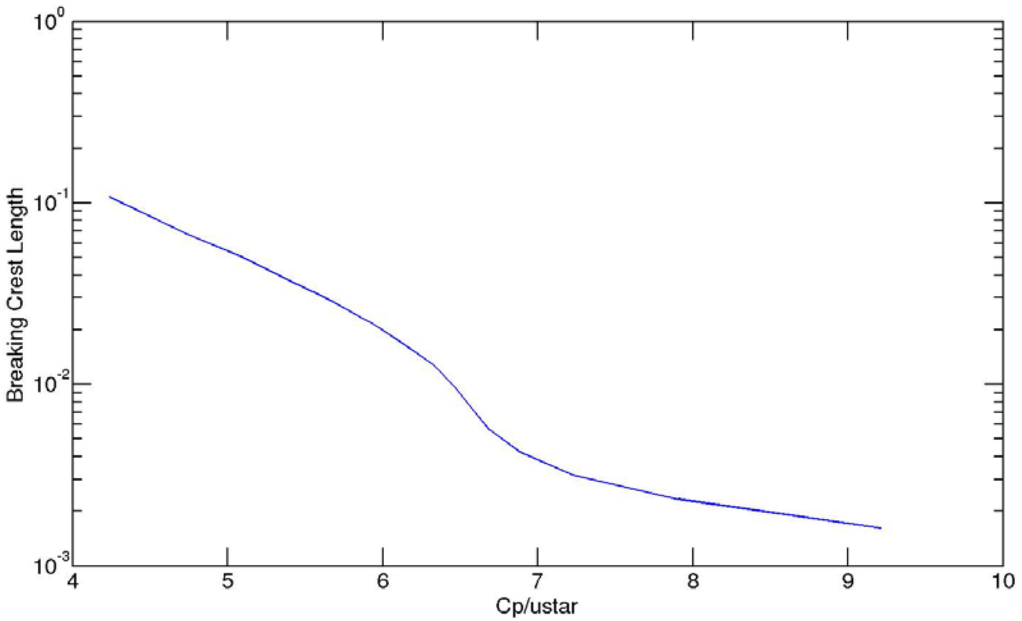
Integrated Source-term, wind = 16.4175 m/s
x 10^-3
2.5
2
1.5
1
0.5
0
Integrated Source tem m^3*s^-3
S_ds
S_in
S_ds OBS
S_dsL
S_in OBS
0
1
2
3
4
5
6
7
8
9
10
Fetch [km]



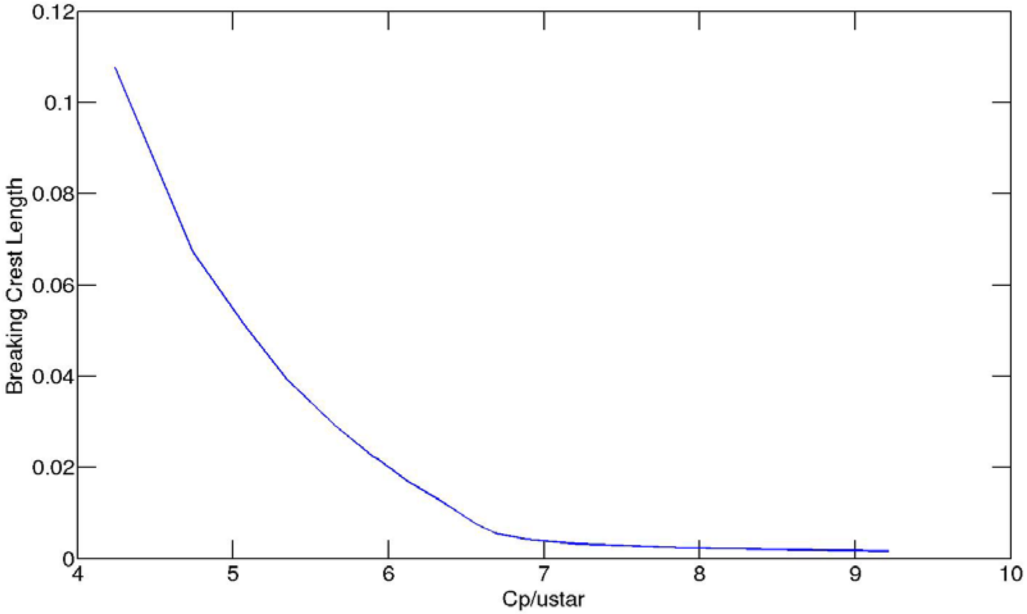
Normalised Saturation
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
1
1.5
2
2.5
3
3.5
4
4.5
5
F/Fp



Normalised Saturation
10^0
10^{-1}
10^{-2}
10^{-3}
10^0
F/Fp



Breaking Crest Length
Cp/ustar



Breaking Crest Length
Cp/ustar

Concluding Remarks

- our framework provides predictions of wave breaking properties (breaking probability, breaking crest length spectral density per unit area and breaking strength) using standard spectral wave models.

- it provides accurate predictions for the limited breaking data available in developing and mature wind seas

- further validation against data will be made as suitable new data sets become available.

- it has been added to existing spectral wave forecasting models. Upgrading the form of the DIA is desirable.

We cant Save Everyone !